Homework 14: Due Wednesday, April 26

Problem 1: Letting f_n be the sequence of Fibonacci numbers (so $f_0 = 0$, $f_1 = 1$, and $f_n = f_{n-1} + f_{n-2}$ for all $n \geq 2$), show that $\gcd(f_n, f_{n+1}) = 1$ for all $n \in \mathbb{N}$.

Note: For each of the counting problems below, you must explain your solution. For example, if your answer is a product, describe the sequence of choices you are making and explain where each term comes from. Numerical answers without written justification will receive no credit.

Problem 2: Using the digits 1 through 9 only (so exclude 0), how many 13 digits numbers are there in which no two consecutive digits are the same?

Problem 3: How many ways are there to pick two distinct cards from a standard 52-card deck such that the first card is a spade and the second is not an ace? In this problem, order matters. So if you pick the 3 of spades followed by the 7 of spades, this is different from the 7 of spades followed by the 3 of spades.

Problem 4: Write an ML program to solve problem 3 by brute force. Start by defining the following types:

```
datatype suit = Club | Diamond | Heart | Spade
datatype rank = Ace | Two | Three | Four | Five | Six | Seven | Eight | Nine | Ten | Jack | Queen | King
type card = rank * suit
```

With these in hand, we can write `val c = (Three, Spade) : card` to form a card object. Now write

```
val allSuits = [Club, Diamond, Heart, Spade]
val allRanks = [Ace, Two, Three, Four, Five, Six, Seven, Eight, Nine, Ten, Jack, Queen, King]
```

to form the set of all suits and the set of all ranks. From here, use only set operations (like `cartProd` and `setFilter`) to create the set of all ordered pairs of cards that satisfy the requirements of Problem 3, and then compute the number of elements in the set.

Problem 5: Suppose that you are creating a password using 26 letters, 10 numbers, and 15 special characters. How many such 10-character passwords are possible if they must have exactly 6 letters, 2 numbers, and 2 special characters?

Problem 6: A local pizza place has three different types of crust, five different meats, and seven different (non meat) toppings. For a given pizza, you can pick any crust, at most 2 meats (so 0, 1, or 2 is possible) and at most 3 toppings (so 0, 1, 2, or 3 is possible). How many pizzas are possible?