

## Written Assignment 1: Due Wednesday, September 7

**Definition:** Let  $a \in \mathbb{Z}$ .

- We say that  $a$  has *type 0* if there exists  $m \in \mathbb{Z}$  with  $a = 4m$ .
- We say that  $a$  has *type 1* if there exists  $m \in \mathbb{Z}$  with  $a = 4m + 1$ .
- We say that  $a$  has *type 2* if there exists  $m \in \mathbb{Z}$  with  $a = 4m + 2$ .
- We say that  $a$  has *type 3* if there exists  $m \in \mathbb{Z}$  with  $a = 4m + 3$ .

Just like for the evens, it is possible to show that every integer is of either type 0, or type 1, or type 2, or type 3. Feel free to use this result in your arguments.

**Problem 1:** Show that for all  $a \in \mathbb{Z}$ , we have that  $a^2$  either has type 0 or has type 1.

*Hint:* Do a proof by cases by considering the type of  $a$ .

**Problem 2:** In this problem, you will prove the following:

“If  $a \in \mathbb{Z}$  and  $a$  has type 0, then there exists  $b, c \in \mathbb{Z}$  with  $a = b^2 - c^2$ ”.

However, we will do it in stages.

- Write down some examples of type 0 integers. For each of these, find examples of  $b$  and  $c$  with  $a = b^2 - c^2$ .
- Looking at your examples, make a guess as to a general pattern. In other words, if we have a type 0 integer  $a$  and we fix  $n \in \mathbb{Z}$  with  $a = 4n$ , what do you guess will work for  $b$  and  $c$ ?
- Now write up a formal proof of the statement.