

Problem Set 1: Due Monday, January 28

Problem 1: For each part, explain your reasoning using a sentence or two.

a. Consider the line in the plane described by the equation $3x - 2y = 12$. Find an example of $a, b, c, d \in \mathbb{R}$ such that

$$\begin{aligned}x &= a + bt \\ y &= c + dt\end{aligned}$$

is a parametric equation for the line.

b. Find two other choices for $a, b, c, d \in \mathbb{R}$ that work for part (a).

c. Consider the line in the plane described parametrically by

$$\begin{aligned}x &= 2 - 3t \\ y &= 1 + 5t.\end{aligned}$$

Find an example of $a, b, c \in \mathbb{R}$ such that the line is described by the equation $ax + by = c$.

d. Find another example of $a, b, c \in \mathbb{R}$ that works for part (c).

Problem 2: Let P be the plane in \mathbb{R}^3 that contains the origin and that is parallel to each of the following two vectors:

$$\vec{u} = \begin{pmatrix} 2 \\ -3 \\ 1 \end{pmatrix} \quad \text{and} \quad \vec{w} = \begin{pmatrix} -7 \\ 1 \\ 4 \end{pmatrix}$$

In the notes, we discussed one way to parametrize P , and we will discuss this in more detail later. Now find an equation of the form $ax + by + cz = d$ for P . Explain your process using a sentence or two.

Problem 3: Let L be the line in \mathbb{R}^3 that is the intersection of the two planes $3x + 4y - z = 2$ and $x - 2y + z = 4$.

a. Using the equations of the planes, determine if the points $(1, 0, 1)$ and $(1, 1, 5)$ are on L .

b. Find a parametric description of L . Explain your process using a sentence or two.

c. Use the parametric description of L to determine if $(5, 2, 3)$ is a point on L . Explain.

Note: Given a point, it seems easier to determine if it is on L using the equations of the planes rather than the parametric description. In contrast, if you want to *generate* points on L , it is easier to use the parametric description (just plug in values for the parameter) than the plane equations.

Problem 4:

a. Do the planes with equations $2x - 3y + z = 7$ and $-4x + 9y - 2z = 3$ intersect? Explain your reasoning.

b. Do the lines described by the two parametric equations

$$\begin{array}{rclcl} x & = & -4 & + & 6t \\ y & = & 2 & + & t \\ z & = & 1 & + & 3t \end{array} \qquad \begin{array}{rclcl} x & = & 4 & + & 4t \\ y & = & 5 & - & t \\ z & = & 9 & - & 2t \end{array}$$

intersect? Explain your reasoning.