

Problem Set 14: Due Monday, April 8

Problem 1: Let $V = \mathbb{R}^3$, but with the following operations:

$$\begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix} + \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

and

$$c \cdot \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix} = \begin{pmatrix} ca_1 \\ ca_2 \\ ca_3 \end{pmatrix}.$$

Show that there is no element of V that serves as $\vec{0}$. That is, show that there does not exist $\vec{z} \in V$ such that $\vec{v} + \vec{z} = \vec{v}$ for all $\vec{v} \in V$.

Problem 2: Let $V = \mathbb{R}^2$, but with the following operations:

$$\begin{pmatrix} a_1 \\ a_2 \end{pmatrix} + \begin{pmatrix} b_1 \\ b_2 \end{pmatrix} = \begin{pmatrix} a_1 + b_1 \\ a_2 + b_2 \end{pmatrix}$$

and

$$c \cdot \begin{pmatrix} a_1 \\ a_2 \end{pmatrix} = \begin{pmatrix} ca_1 \\ a_2 \end{pmatrix}.$$

Also, let

$$\vec{0} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}.$$

Show that V is not a vector space by explicitly finding a counterexample to one of the 10 properties.

Problem 3: Let V be a vector space. Show that $\vec{u} + (\vec{v} + \vec{w}) = \vec{w} + (\vec{v} + \vec{u})$ for all $\vec{u}, \vec{v}, \vec{w} \in V$. Carefully state what property you are using in every step of your argument.

Problem 4: Show that

$$\left\{ \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix} \in \mathbb{R}^3 : a_1 + a_2 + a_3 = 0 \right\}$$

is a subspace of \mathbb{R}^3 .

Problem 5: Recall that \mathcal{P} is the vector space of all polynomial functions $f: \mathbb{R} \rightarrow \mathbb{R}$. Let W be the subset of \mathcal{P} consisting of those polynomials that have a nonnegative constant term (i.e. the constant term is greater than or equal to 0). Is W a subspace of \mathcal{P} ? Either prove or give a counterexample.

Problem 6: Let V be the vector space of all 2×2 matrices. Show that

$$\begin{pmatrix} -2 & 7 \\ -1 & -9 \end{pmatrix} \in \text{Span} \left(\begin{pmatrix} 1 & 1 \\ 2 & -3 \end{pmatrix}, \begin{pmatrix} 3 & 0 \\ 5 & -4 \end{pmatrix} \right).$$