

## Writing Assignment 6: Due Wednesday, March 13

**Problem 1:** Suppose that  $A$  is an  $2 \times 2$  idempotent matrix (recall this means that  $A^2 = A$ ).

- Show that the only possible eigenvalues of  $A$  are 0 and 1.
- Show that if  $A \neq I$ , then  $A$  is not invertible.
- Show that if  $A \neq I$ , then 0 must be an eigenvalue of  $A$ .

**Problem 2:** Let  $T: \mathbb{R}^2 \rightarrow \mathbb{R}^2$  be a linear transformation. Show that  $\text{Null}(T) \neq \{\vec{0}\}$  if and only if there exists a basis  $\alpha$  of  $\mathbb{R}^2$  such that the second column of  $[T]_\alpha$  is the zero vector.

*Note:* Recall that “if and only if” means that each of the two directions are true, so you need to argue both.