

Homework 21 : Due Monday, November 9

Problem 1: Chapter 12, #2ab

Problem 2: Chapter 12, #3

Problem 3: Chapter 12, #5ab

Hint: Use a bit of theory to help with the computations. You know the orbits partition X , and you know that the size of any orbit divides $|G|$. Once you know \mathcal{O}_x , you can use the Orbit-Stabilizer Theorem to determine $|G_x|$.

Problem 4: Let $G = \mathbb{R}$ (under addition) and let $X = \mathbb{R}^2$. Define a function from $G \times X$ to X by $a * (x, y) = (x + ay, y)$.

- Show that $*$ is an action of G on X .
- Describe the orbits of the action geometrically.

Problem 5: Let $G = S_3$ and let

$$X = \{1, 2, 3\} \times \{1, 2, 3\} = \{(1, 1), (1, 2), (1, 3), (2, 1), (2, 2), (2, 3), (3, 1), (3, 2), (3, 3)\}$$

Define a function from $G \times X$ to X by $\sigma * (x, y) = (\sigma(x), \sigma(y))$.

- Show that $*$ is an action of G on X .
- Find the orbits of the action.